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**GATE SOLVED PAPER**  
**Mathematics**  
**2004**

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# GATE SOLVED PAPER - MA

2004

## Q. 1 - Q. 30 Carry one mark each.

**The symbols  $N, Z, Q$  and  $R$  denote the set of natural numbers, integers, rational numbers and real numbers respectively, throughout the paper.**

- Q. 1 Let  $S$  and  $T$  be two subspaces of  $R^{24}$  such that  $\dim(S) = 19$  and  $\dim(T) = 17$ . Then, the  
(A) smallest possible value of  $\dim(S \cap T)$  is 2  
(B) Largest possible value of  $\dim(S \cap T)$  is 18  
(C) smallest possible value of  $\dim(S + T)$  is 19  
(D) largest possible value of  $\dim(S + T)$  is 22
- Q. 2 Let  $v_1 = (1, 2, 0, 3, 0)$ ,  $v_2 = (1, 2, -1, -1, 0)$ ,  $v_3 = (0, 0, 1, 4, 0)$ ,  $v_4 = (2, 4, 1, 10, 1)$ , and  $v_5 = (0, 0, 0, 0, 1)$ . The dimension of the linear span of  $\{v_1, v_2, v_3, v_4, v_5\}$  is  
(A) 2 (B) 3  
(C) 4 (D) 5
- Q. 3 The set  $V = \{(x, y) \in R^2 : xy \geq 0\}$  is  
(A) a vector subspace of  $R^2$   
(B) Not a vector subspace of  $R^2$  since every element does not have an inverse in  $V$   
(C) not a vector subspace of  $R^2$  since it is not closed under scalar multiplication  
(D) not a vector subspace of  $R^2$  since it is not closed under vector addition
- Q. 4 Let  $f: R^4 \rightarrow R$  be a linear functional defined by  $f(x_1, x_2, x_3, x_4) = -x_2$ . If  $\langle \cdot, \cdot \rangle$  denotes the standard inner product on  $R^4$ , then the unique vector  $v \in R^4$  such that  $f(w) = \langle v, w \rangle$  for all  $w \in R^4$ , is  
(A)  $(0, -1, 0, 0)$  (B)  $(-1, 0, -1, 1)$   
(C)  $(0, 1, 0, 0)$  (D)  $(1, 0, 1, -1)$
- Q. 5 If  $D$  is the open unit disk in  $C$  and  $f: C \rightarrow D$  is analytic with  $f(10) = 1/2$ , then  $f(10 + i)$  is  
(A)  $\frac{1+i}{2}$  (B)  $\frac{1-i}{2}$   
(C)  $\frac{1}{2}$  (D)  $\frac{i}{2}$
- Q. 6 The real part of the principal value of  $4^{4-i}$  is  
(A)  $256 \cos(\ln 4)$  (B)  $64 \cos(\ln 4)$   
(C)  $16 \cos(\ln 4)$  (D)  $4 \cos(\ln 4)$
- Q. 7 If  $\sin z = \sum_{n=0}^{\infty} a_n (z - \pi/4)^n$ , then  $a_6$  equals

- (A) 0 (B)  $\frac{1}{720}$   
 (C)  $\frac{1}{(720\sqrt{2})}$  (D)  $\frac{-1}{(720\sqrt{2})}$
- Q. 8 The equation  $x^6 - x - 1 = 0$  has  
 (A) no positive real roots (B) exactly one positive real root  
 (C) exactly two positive real roots (D) all positive real roots
- Q. 9 Let  $f, g: (0,1) \times (0,1) \rightarrow R$  be two continuous functions defined by  
 $f(x,y) = \frac{1}{1+x(1-y)}$  and  $g(x,y) = \frac{1}{1+x(y-1)}$ . Then, on  $(0,1) \times (0,1)$ ,  
 (A)  $f$  and  $g$  are both uniformly continuous  
 (B)  $f$  is uniformly continuous but  $g$  is not  
 (C)  $g$  is uniformly continuous but  $f$  is not  
 (D) neither  $f$  nor  $g$  is uniformly continuous
- Q. 10 Let  $S$  be the surface bounding the region  $x^2 + y^2 \leq 1, x \geq 0, y \geq 0, z \leq 1$ , and  
 $\hat{n}$  be the unit outer normal to  $S$ . Then  $\iint_S [(\sin^2 x)\hat{i} + 2y\hat{j} - z(1 + \sin 2x)\hat{k}] \cdot \hat{n} dS$   
 equals  
 (A) 1 (B)  $\frac{\pi}{2}$   
 (C)  $\pi$  (D)  $2\pi$
- Q. 11 Let  $f: [0, \infty) \rightarrow R$  be defined by  

$$f(x) = \begin{cases} \frac{1}{\sqrt{x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$
  
 Consider the two improper integrals  $I_1 = \int_0^1 f(x) dx$  and  $I_2 = \int_1^\infty f(x) dx$ . Then  
 (A) both  $I_1$  and  $I_2$  exist (B)  $I_1$  exists but  $I_2$  does not  
 (C)  $I_1$  does not exist but  $I_2$  does (D) neither  $I_1$  nor  $I_2$  exists
- Q. 12 The orthogonal trajectories to the family of straight lines  $y = k(x-1), k \in R$ ,  
 are given by  
 (A)  $(x-1)^2 + (y-1)^2 = c^2$  (B)  $x^2 + y^2 = c^2$   
 (C)  $x^2 + (y-1)^2 = c^2$  (D)  $(x-1)^2 + y^2 = c^2$
- Q. 13 If  $y = \varphi(x)$  is a particular solution of  $y'' + (\sin x)y' + 2y = e^x$  and  $y = \Psi(x)$  is a  
 particular solution of  $y'' + (\sin x)y' + 2y = \cos 2x$ , then a particular solution of  
 $y'' + (\sin x)y' + 2y = e^x + 2\sin^2 x$ , is given by  
 (A)  $\varphi(x) - \Psi(x) + \frac{1}{2}$  (B)  $\Psi(x) - \varphi(x) + \frac{1}{2}$   
 (C)  $\varphi(x) - \Psi(x) + 1$  (D)  $\Psi(x) - \varphi(x) + 1$
- Q. 14 Let  $P_n(x)$  be the Legendre polynomial of degree  $n \leq 10$ . If  $1 + x^{10} = \sum_{n=0}^{10} c_n P_n(x)$ ,  
 then  $c_5$  equals  
 (A) 0 (B)  $\frac{2}{11}$   
 (C) 1 (D)  $\frac{11}{2}$

- Q. 15 Let  $I$  be the set of irrational real numbers and let  $G = I \cup \{0\}$ . Then, under the usual addition of real numbers,  $G$  is  
(A) a group, since  $R$  and  $Q$  are groups under addition  
(B) a group, since the additive identity is in  $G$   
(C) not a group, since addition on  $G$  is not a binary operation  
(D) not a group, since not all elements in  $G$  have an inverse
- Q. 16 In the group  $(Z, +)$ , the subgroup generated by 2 and 7 is  
(A)  $Z$  (B)  $5Z$   
(C)  $9Z$  (D)  $14Z$
- Q. 17 The cardinality of the center of  $Z_{12}$  is  
(A) 1 (B) 2  
(C) 3 (D) 12
- Q. 18 Suppose  $X = (1, \infty)$  and  $T: X \rightarrow X$  is such that  $d(Tx, Ty) < d(x, y)$  for  $x \neq y$ . Then  
(A)  $T$  has at most one fixed point  
(B)  $T$  has a unique fixed point, by Banach Contraction Theorem  
(C)  $T$  has infinitely many fixed points  
(D) for every  $x \in X, \{T^n(x)\}$  converges to a fixed point
- Q. 19 Consider  $R^2$  with  $\| \cdot \|_1$  norm and  $M = \{(x, 0) : x \in R\}$ . Define  $g: M \rightarrow R$  by  $g(x, y) = x$ . Then a Hahn-Banach extension  $f$  of  $g$  is given by  
(A)  $f(x, y) = 2x$  (B)  $f(x, y) = x + y$   
(C)  $f(x, y) = x - 2y$  (D)  $f(x, y) = x + 2y$
- Q. 20 Let  $X$  be an inner product space and  $S \subset X$ . Then it follows that  
(A)  $S^\perp$  has nonempty interior (B)  $S^\perp = \{0\}$   
(C)  $S^\perp$  is a closed subspace (D)  $(S^\perp)^\perp = S$
- Q. 21 An iterative scheme is given by  $x_{n+1} = \frac{1}{5} \left( 16 - \frac{12}{x_n} \right)$ ,  $n \in N \cup \{0\}$ . Such a scheme, with suitable  $x_0$ , will  
(A) not converge (B) converge to 1.6  
(C) converge to 1.8 (D) converge to 2
- Q. 22 In the  $(x, t)$  plane, the characteristics of the initial value problem  $u_t + uu_x = 0$ , with  $u(x, 0) = x$ ,  $0 \leq x \leq 1$ , are  
(A) parallel straight lines  
(B) straight lines which intersect at  $(0, -1)$   
(C) non-intersecting parabolas  
(D) concentric circles with centre at the origin
- Q. 23 Suppose  $u(x, y)$  satisfies Laplace's equation:  $\nabla^2 u = 0$  in  $R^2$  and  $u = x$  on the unit circle. Then, at the origin  
(A)  $u$  tends to infinity (B)  $u$  attains a finite minimum  
(C)  $u$  attains a finite maximum (D)  $u$  is equal to 0

- Q. 24 A circular disk of radius  $a$  and mass  $m$  is supported on a needle at its centre. The disk is set spinning with initial angular velocity  $\omega_0$  about an axis making an angle  $\pi/6$  with the normal to the disk. If  $\bar{\omega}(t)$  is the angular velocity of the disk at any time  $t$ , then its component along the normal equals
- (A)  $\frac{\sqrt{3}\omega_0}{2}$  (B)  $\omega_0$   
 (C)  $\omega_0 \sin t$  (D)  $\left(\frac{\sqrt{3}\omega_0}{2}\right) \cos t$
- Q. 25 In  $R^2$  with usual topology, the set  $U = \{(x, -y) \in R^2 : x = 0, 1, -1 \text{ and } y \in N\}$  is
- (A) neither closed nor bounded (B) closed but not bounded  
 (C) bounded but not closed (D) closed and bounded
- Q. 26 In  $R^3$  with usual topology, let  $V = \{(x, y, z) \in R^3 : x^2 + y^2 + z^2 = 1, y \neq 0\}$  and  $W = \{(x, y, z) \in R^3 : y = 0\}$ . Then  $V \cup W$  is
- (A) connected and compact (B) connected but not compact  
 (C) compact but not connected (D) neither connected nor compact
- Q. 27 Suppose  $X$  is a random variable,  $c$  is a constant and  $a_n = E(X - c)^n$  is finite for all  $n \geq 1$ . Then  $P(X = c) = 1$  if and only if  $a_n = 0$  for
- (A) at least one  $n \geq 1$  (B) at least one odd  $n$   
 (C) at least one even  $n$  (D) at least two values of  $n$
- Q. 28 If the random vector  $(X_1, X_2)^T$  has a bivariate normal distribution with mean vector  $(\mu, \mu)^T$  and the matrix  $(E(X_i X_j))_{1 \leq i, j \leq 2}$  equals  $\begin{pmatrix} \alpha_1 & \mu^2 \\ \mu^2 & \alpha_2 \end{pmatrix}$ , where  $\mu \in R$  and  $\alpha_1, \alpha_2 > \mu^2$ , then  $X_1$  and  $X_2$  are
- (A) independent for all  $\alpha_1$  and  $\alpha_2$   
 (B) independent if and only if  $\alpha_1 = \alpha_2$   
 (C) uncorrelated, but not independent for all  $\alpha_1, \alpha_2$   
 (D) uncorrelated if and only if  $\alpha_1 = \alpha_2$  and in this case they are not independent
- Q. 29 If the cost matrix for an assignment problem is given by
- $$\begin{pmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{pmatrix}$$
- where  $a, b, c, d > 0$ , then the value of the assignment problem is
- (A)  $a + b + c + d$  (B)  $\min\{a, b, c, d\}$   
 (C)  $\max\{a, b, c, d\}$  (D)  $4 \min\{a, b, c, d\}$
- Q. 30 Extremals for the variational problem  $v[y(x)] = \int_1^2 (y^2 + x^2 y^2) dx$  satisfy the differential equation
- (A)  $x^2 y'' + 2ny' - y = 0$  (B)  $x^2 y'' - 2xy' + y = 0$   
 (C)  $2xy' - y = 0$  (D)  $x^2 y'' - y = 0$

**Q. 31 - Q. 90 Carry two marks each.**

- Q. 31 Let  $V$  be the subspace of  $R^3$  spanned by  $u = (1, 1, 1)$  and  $v = (1, 1, -1)$ . The

orthonormal basis of  $V$  obtained by the Gram-Schmidt process on the ordered basis  $(u, v)$  of  $V$  is

(A)  $\left\{ \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left( \frac{2}{3}, \frac{2}{3}, -\frac{4}{3} \right) \right\}$

(B)  $\{(1, 1, 0), (1, 0, 1)\}$

(C)  $\left\{ \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left( \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}} \right) \right\}$

(D)  $\left\{ \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right), \left( \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right) \right\}$

- Q. 32 In  $R^2$ ,  $\langle (x_1, y_1), (x_2, y_2) \rangle = x_1 x_2 - \alpha(x_2 y_1 + x_1 y_2) + y_1 y_2$  is an inner product  
 (A) for all  $\alpha \in R$  (B) if and only if  $\alpha = 0$   
 (C) if and only if  $\alpha < 1$  (D) if and only if  $\alpha < 1$
- Q. 33 Let  $\{v_1, v_2, v_3, v_4\}$  be a basis of  $R^4$  and  $v = a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4$ , where  $a_i \in R$ ,  $i = 1, 2, 3, 4$ . Then  $\{v_1 - v, v_2 - v, v_3 - v, v_4 - v\}$  is a basis of  $R^4$  if and only if  
 (A)  $a_1 = a_2 = a_3 = a_4$  (B)  $a_1 a_2 a_3 a_4 = -1$   
 (C)  $a_1 + a_2 + a_3 + a_4 \neq 0$  (D)  $a_1 + a_2 + a_3 + a_4 \neq 0$
- Q. 34 Let  $R^{2 \times 2}$  be the real vector space of all  $2 \times 2$  real matrices. For  $Q = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$ , define a linear transformation  $T$  on  $R^{2 \times 2}$  as  $T(P) = QP$ . Then the rank of  $T$  is  
 (A) 1 (B) 2  
 (C) 3 (D) 4
- Q. 35 Let  $P$  be a  $n \times n$  matrix with integral entries and  $Q = P + \frac{I}{2}$ , where  $I$  denotes the  $n \times n$  identity matrix. Then  $Q$  is  
 (A) idempotent, i.e.  $Q^2 = Q$   
 (B) invertible  
 (C) nilpotent  
 (D) unipotent, i.e.,  $Q - I$  is nilpotent
- Q. 36 Let  $M$  be a square matrix of order, 2 such that rank of  $M$  is 1. Then  $M$  is  
 (A) diagonalizable and nonsingular  
 (B) diagonalizable and nilpotent  
 (C) neither diagonalizable nor nilpotent  
 (D) either diagonalizable or nilpotent but not both
- Q. 37 If  $M$  is a  $7 \times 5$  matrix of rank 3 and  $N$  is a  $5 \times 7$  matrix of rank 5, then rank  $(MN)$  is  
 (A) 5 (B) 3  
 (C) 2 (D) 1
- Q. 38  $\int_0^{2\pi} \frac{d\theta}{13 - 5 \sin \theta} =$   
 (A)  $-\frac{\pi}{6}$  (B)  $-\frac{\pi}{12}$   
 (C)  $\frac{\pi}{12}$  (D)  $\frac{\pi}{6}$

- Q. 39 In the Laurent series expansion of  $f(z) = \frac{1}{z-1} - \frac{1}{z-2}$  valid in the region  $|z| > 2$ , the coefficient of  $\frac{1}{z^2}$  is
- (A) -1 (B) 0  
(C) 1 (D) 2
- Q. 40 Let  $w = f(z)$  be the bilinear transformation that maps  $-1, 0$  and  $1$  to  $-i, 1$  and  $i$  respectively. Then  $f(1-i)$  equals
- (A)  $-1 + 2i$  (B)  $2i$   
(C)  $-2 + i$  (D)  $-1 + i$
- Q. 41 For the positively oriented unit circle,  $\oint_{|z|=1} \frac{2\operatorname{Re}(z)}{z+2} dz =$
- (A) 0 (B)  $\pi i$   
(C)  $2\pi i$  (D)  $4\pi i$
- Q. 42 The number of zeroes, counting multiplicities, of the polynomial  $z^5 + 3z^3 + z^2 + 1$  inside the circle  $|z| = 2$  is
- (A) 0 (B) 2  
(C) 3 (D) 5
- Q. 43 Let  $f = u + iv$  and  $g = v + iw$  be non-zero analytic functions on  $|z| < 1$ . Then it follows that
- (A)  $f' \equiv 0$  (B)  $f$  is conformal on  $|z| < 1$   
(C)  $f \equiv kg$  for some real  $k$  (D)  $f$  is one to one
- Q. 44 If  $f(x, y) = \begin{cases} x^3/(x^2 + y^2) & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ , then at  $(0, 0)$
- (A)  $f_x, f_y$  do not exist  
(B)  $f_x, f_y$  exist and are equal  
(C) the directional derivative exists along any straight line  
(D)  $f$  is differentiable
- Q. 45 Let  $\alpha > 1$  and  $g(x) = \sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ ,  $\sigma \leq x < \infty$ . Then  $g(x)$  is
- (A) not continuous  
(B) continuous but not differentiable  
(C) differentiable but not continuously differentiable  
(D) continuously differentiable
- Q. 46 The sequence of functions  $\{f_n\}$  on  $[0, 1]$  with Lebesgue measure, defined by  $f_n(x) = \begin{cases} x, & 0 \leq x < 1 - 1/n \\ \sqrt{n}, & 1 - 1/n \leq x \leq 1 \end{cases}$ , converges
- (A) almost everywhere and as well as in  $L^1$   
(B) almost everywhere but not in  $L^1$   
(C) in  $L^1$ , but not almost everywhere  
(D) neither almost everywhere nor in  $L^1$

- Q. 47 Consider two sequences  $\{f_n\}$  and  $\{g_n\}$  of functions where  $f_n: [0,1] \rightarrow R$  and  $g_n: R \rightarrow R$  are defined by

$$f_n(x) = x^n \text{ and } g_n(x) = \begin{cases} \cos(x-n)\pi/2 & \text{if } x \in [n-1, n+1] \\ 0 & \text{otherwise} \end{cases}. \text{ Then}$$

- (A) neither  $\{f_n\}$  nor  $\{g_n\}$  is uniformly convergent  
 (B)  $\{f_n\}$  is not uniformly convergent but  $\{g_n\}$  is  
 (C)  $\{g_n\}$  is not uniformly convergent but  $\{f_n\}$  is  
 (D) both  $\{f_n\}$  and  $\{g_n\}$  are uniformly convergent

- Q. 48 Let  $f: [0,1] \rightarrow R$  and  $g: [0,1] \rightarrow R$  be two functions defined by

$$f(x) = \begin{cases} \frac{1}{n} & \text{if } x = \frac{1}{n}, n \in N \\ 0 & \text{otherwise} \end{cases} \text{ and } g(x) = \begin{cases} n & \text{if } x = \frac{1}{n}, n \in N \\ 0 & \text{otherwise} \end{cases}. \text{ Then}$$

- (A) both  $f$  and  $g$  are Riemann integrable  
 (B)  $f$  is Riemann integrable but  $g$  is not  
 (C)  $g$  is Riemann integrable but  $f$  is not  
 (D) neither  $f$  nor  $g$  is Riemann integrable

- Q. 49 The set of all continuous functions  $f: [0,1] \rightarrow R$  satisfying

$$\int_0^1 t^n f(t) dt = 0, \quad n = 0, 1, 2, \dots$$

- (A) is empty  
 (B) contains a single element  
 (C) is countably infinite  
 (D) is uncountably infinite

- Q. 50 Let  $f: R^3 \rightarrow R^3$  be defined by  $f(x_1, x_2, x_3) = (x_2 + x_3, x_3 + x_1, x_1 + x_2)$ . Then the first derivative of  $f$  is

- (A) not invertible anywhere  
 (B) invertible only at the origin  
 (C) invertible everywhere except at the origin  
 (D) invertible everywhere

- Q. 51 Let  $y = \varphi(x)$  and  $y = \Psi(x)$  be solutions of  $y'' - 2xy' + (\sin x^2)y = 0$  such that  $\varphi(0) = 1$ ,  $\varphi'(0) = 1$  and  $\Psi(0) = 1$ ,  $\Psi'(0) = 2$ . Then the value of the Wronskian  $W(\varphi, \Psi)$  at  $x = 1$  is

- (A) 0  
 (B) 1  
 (C)  $e$   
 (D)  $e^2$

- Q. 52 The set of all eigenvalues of the Sturm-Liouville problem

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'\left(\frac{\pi}{2}\right) = 0,$$

is given by

- (A)  $\lambda = 2n, n = 1, 2, 3, \dots$   
 (B)  $\lambda = 2n, n = 0, 1, 2, 3, \dots$   
 (C)  $\lambda = 4n^2, n = 1, 2, 3, \dots$   
 (D)  $\lambda = 4n^2, n = 0, 1, 2, 3, \dots$



- Q. 53 If  $Y(p)$  is the Laplace transform of  $y(t)$ , which is the solution of the initial value problem  $\frac{d^2 y}{dt^2} + y(t) = \begin{cases} 0, & 0 < t < 2\pi \\ \sin t, & t > 2\pi \end{cases}$ , with  $y(0) = 1$  and  $y'(0) = 0$ , then  $Y(p)$  equals
- (A)  $\frac{p}{1+p^2} + \frac{e^{-2\pi p}}{(1+p^2)^2}$  (B)  $\frac{p+1}{1+p^2}$   
 (C)  $\frac{p}{1+p^2} + \frac{e^{-2\pi p}}{(1+p^2)}$  (D)  $\frac{p(1+p^2)+1}{(1+p^2)^2}$
- Q. 54 If  $y = \sum_{m=0}^{\infty} a_m x^m$  is a solution of  $y'' + xy' + 3y = 0$ , then  $\frac{a_m}{a_{m+2}}$  equals
- (A)  $\frac{(m+1)(m+2)}{m+3}$  (B)  $-\frac{(m+1)(m+2)}{m+3}$   
 (C)  $-\frac{m(m-1)}{m+3}$  (D)  $\frac{m(m-1)}{m+3}$
- Q. 55 The indicial equation for :  $x(1+x^2)y'' + (\cos x)y' + (1-3x+x^2)y = 0$ , is
- (A)  $r^2 - r = 0$  (B)  $r^2 + r = 0$   
 (C)  $r^2 = 0$  (D)  $r^2 - 1 = 0$
- Q. 56 The general solution  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$  of the system
- $$\begin{aligned} x &= -x + 2y \\ y &= 4x + y \end{aligned}$$
- is given by
- (A)  $\begin{pmatrix} c_1 e^{3t} - c_2 e^{-3t} \\ 2c_1 e^{3t} + c_2 e^{-3t} \end{pmatrix}$  (B)  $\begin{pmatrix} c_1 e^{3t} \\ c_2 e^{-3t} \end{pmatrix}$   
 (C)  $\begin{pmatrix} c_1 e^{3t} + c_2 e^{-3t} \\ 2c_1 e^{3t} + c_2 e^{-3t} \end{pmatrix}$  (D)  $\begin{pmatrix} c_1 e^{3t} - c_2 e^{-3t} \\ -2c_1 e^{3t} + c_2 e^{-3t} \end{pmatrix}$
- Q. 57 Let  $G$  and  $H$  be two groups. The groups  $G \times H$  and  $H \times G$  are isomorphic
- (A) for any  $G$  and any  $H$  (B) only if one of them is cyclic  
 (C) only if one of them is abelian (D) only if  $G$  and  $H$  are isomorphic
- Q. 58 Let  $H = Z_2 \times Z_6$  and  $K = Z_2 \times Z_4$ . Then
- (A)  $H$  is isomorphic to  $K$  since both are cyclic  
 (B)  $H$  is isomorphic to  $K$  since 2 divides 6 and g.c.d. (3,4) = 1  
 (C)  $H$  is not isomorphic to  $K$  since  $K$  is cyclic whereas  $H$  is no  
 (D)  $H$  is not isomorphic to  $K$  since there is no homomorphism from  $H$  to  $K$
- Q. 59 Suppose  $G$  denotes the multiplicative group  $\{-1, 1\}$  and  $S = \{z \in C : |z| = 1\}$ . Let  $G$  act on  $S$  by complex multiplication. Then the cardinality of the orbit of  $i$  is
- (A) 1 (B) 2  
 (C) 5 (D) infinite
- Q. 60 The number of 5-Sylow subgroups of  $Z_{20}$  is
- (A) 1 (B) 4  
 (C) 5 (D) 6

Q. 61 Let  $S = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in R \right\}$  be the ring under matrix addition and multiplication.

Then the subset  $\left\{ \begin{pmatrix} 0 & p \\ 0 & 0 \end{pmatrix} : p \in R \right\}$  is

- (A) not an ideal of  $S$
- (B) an ideal but not a prime ideal of  $S$
- (C) is a prime ideal but not a maximal ideal of  $S$
- (D) is a maximal ideal of  $S$

Q. 62 Consider  $S = C[x^5]$ , complex polynomials is  $x^5$ , as a subset of  $T = C[x]$ , the ring of all complex polynomials. Then

- (A)  $S$  is neither an ideal nor a subring of  $T$
- (B)  $S$  is an ideal, but not a subring of  $T$
- (C)  $S$  is a subring but not an ideal of  $T$
- (D)  $S$  is both a subring and an ideal to  $T$

Q. 63 Which of the following statements is true about  $S = Z[x]$ ?

- (A)  $S$  is an Euclidean domain since all its ideals are principal
- (B)  $S$  is an Euclidean domain since  $Z$  is an Euclidean domain
- (C)  $S$  is not an Euclidean domain since  $S$  is not even an integral domain
- (D)  $S$  is not an Euclidean domain since it has non-principal ideals

Q. 64 Let  $X$  be the space of bounded real sequences with sup norm. Define a linear operator  $T : X \rightarrow X$  by

$$T(x) = \left( \frac{x_1}{1}, \frac{x_2}{2}, \dots \right) \text{ for } x = (x_1, x_2, \dots) \in X. \text{ Then}$$

- (A)  $T$  is bounded but not one to one
- (B)  $T$  is one to one but not bounded
- (C)  $T$  is bounded and its inverse (from range of  $T$ ) exists but is not bounded
- (D)  $T$  is bounded and its inverse (from range of  $T$ ) exists and is bounded

Q. 65 Let  $X$  be the space of real sequences having finitely many non-zero terms with  $\| \cdot \|_p, 1 \leq p \leq \infty$ . Then

- (A)  $f$  is continuous only for  $p = 1$
- (B)  $f$  is continuous only for  $p = 2$
- (C)  $f$  is continuous only for  $p = \infty$
- (D)  $f$  is not continuous for any  $p, 1 \leq p \leq \infty$

Q. 66 Let  $X = C^1[0,1]$  with the norm  $\|x\| = \|x\|_\infty + \|x'\|_\infty$  (where  $x'$  is the derivative of  $x$ ) and  $Y = C^1[0,1]$  with sup norm. If  $T$  is the identity operator from  $X$  into  $Y$ , then

- (A)  $T$  and  $T^{-1}$  are continuous
- (B)  $T$  is continuous but  $T^{-1}$  is not
- (C)  $T^{-1}$  is continuous but  $T$  is not
- (D) neither  $T$  nor  $T^{-1}$  is continuous

Q. 67 Let  $X = C[-1,1]$  with the inner product defined by

$$\langle x, y \rangle = \int_{-1}^1 x(t)y(t)dt$$

Let  $Y$  be the set of all odd functions in  $X$ . Then

- (A)  $Y^\perp$  is the set of all even functions in  $X$
- (B)  $Y^\perp$  is the set of odd functions in  $X$
- (C)  $Y^\perp = \{0\}$
- (D)  $Y^\perp$  is the set of all constant functions in  $X$

Q. 68 Let  $X = l^2$ , then space of all square-summable sequences with

$$\|x\| = \sqrt{\sum_{i=1}^{\infty} x_i^2}, \text{ for } x = (x_i) \in X$$

Define a sequence  $\{T_n\}$  of linear operators on  $X$  by  $T_n(x) = (x_1, x_2, \dots, x_n, 0, 0, \dots)$ . Then

- (A)  $T_n$  is an unbounded operator for sufficiently large  $n$
- (B)  $T_n$  is bounded but not compact for all  $n$
- (C)  $T_n$  is compact for all  $n$  but  $\lim_{n \rightarrow \infty} T_n$  is not compact
- (D)  $T_n$  is compact for all  $n$  and so is  $\lim_{n \rightarrow \infty} T_n$

Q. 69 To find the positive square root of  $a > 0$  by solving  $x^2 - a = 0$  by the Newton-Raphson method, if  $x_n$  denotes the  $n^{\text{th}}$  iterate with  $x_0 > 0$ ,  $x_0 \neq \sqrt{a}$ , then the sequence  $\{x_n, n \geq 1\}$  is

- (A) strictly decreasing
- (B) strictly increasing
- (C) constant
- (D) not convergent

Q. 70 In solving the ordinary differential equation  $y' = 2x$ ,  $y(0) = 0$  using Euler's method, the iterates  $y_n$ ,  $n \in N$  satisfy

- (A)  $y_n = x_n^2$
- (B)  $y_n = 2x_n$
- (C)  $y_n = x_n x_{n-1}$
- (D)  $y_n = x_{n-1} + x_n$

Q. 71 The characteristic curves of the partial differential equation

$$(2x + u)u_x + (2y + u)u_y = u,$$

passing through  $(1, 1)$  for any arbitrary initial values prescribed on a non-characteristic curve are given by

- (A)  $x = y$
- (B)  $x^2 + y^2 = 2$
- (C)  $x + y = 2$
- (D)  $x^2 - xy + y^2 = 1$

Q. 72 The solution of Laplace's equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$$

in the unit disk with boundary conditions  $u(1, \theta) = 2 \cos^2 \theta$  is given by

- (A)  $1 + r^2 \cos 2\theta$
- (B)  $1 + \ln r + r \cos 2\theta$
- (C)  $2r^3 \cos^2 \theta$
- (D)  $1 - r^2 + 2r^2 \cos^2 \theta$

Q. 73 For the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ on } R \times [0, T], \text{ with } u(x, 0) = u_0(x), u_0 \in L^2(R),$$

- (A) the solution is reversible in time
- (B) if  $u_0(x)$  has compact support, so does  $u(x, t)$  for any given  $t$
- (C) if  $u_0(x)$  is discontinuous at a point, so is  $u(x, t)$  for any given  $t$
- (D) if  $u_0(x) \geq 0$  for all  $x$ , then  $u(x, t) \geq 0$  for all  $x$  and  $t > 0$

- Q. 74 If  $u(x, t)$  satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad x \in R, \quad t > 0, \quad \text{with initial conditions}$$

$$u(x, 0) = \begin{cases} \sin \frac{\pi x}{c} & 0 \leq x \leq c \\ 0 & \text{elsewhere} \end{cases}, \quad \text{and } u_t(x, 0) = 0 \text{ for all } x,$$

then for a given  $t > 0$ ,

- (A) there are values of  $x$  at which  $u(x, t)$  is discontinuous  
 (B)  $u(x, t)$  is continuous, but  $x_x(x, t)$  is not continuous  
 (C)  $u(x, t)$ ,  $u_x(x, t)$  are continuous, but  $u_{xx}(x, t)$  is not continuous  
 (D)  $u(x, t)$  is smooth for all  $x$
- Q. 75 A rigid body is acted on by two forces,  $F_1 = a\hat{i} + b\hat{j} - 3\hat{k}$  at the point  $(1, 2, -1)$  and  $F_2 = \hat{i} + a\hat{j} + b\hat{k}$  at the point  $(-1, 0, 1)$ . If the force system is equipollent to the force  $F$  and the couple  $G$ , which have no components along  $\hat{k}$ , then  $F$  equals  
 (A)  $2\hat{i} + 4\hat{j}$  (B)  $2\hat{i} - 4\hat{j}$   
 (C)  $4\hat{i} + 2\hat{j}$  (D)  $4\hat{i} - 2\hat{j}$
- Q. 76 A frictionless wire, fixed at  $R$ , rotates with constant angular velocity  $\omega$  about a vertical axis  $RO$  ( $O$  is the origin and  $R$  is above  $O$ ), making a constant angle  $\alpha$  with it. A particle  $P$  of unit mass is constrained to move on the wire. If the mass of the wire is negligible, distance  $OR$  is  $h$  and  $RP$  is  $r(t)$  at any time  $t$ , then the Lagrangian of the motion is  
 (A)  $\frac{1}{2}r^2 - g(h - r \cos \alpha)$  (B)  $\frac{1}{2}(r^2 + \omega^2 r^2) + g r \cos \alpha$   
 (C)  $\frac{1}{2}(r^2 + \omega^2 r^2 \sin^2 \alpha) - g(h - r \cos \alpha)$  (D)  $\frac{1}{2}(r^2 + r^2 \sin^2 \alpha) - gh$
- Q. 77 In  $R$  with the usual topology, the set  $U = \{x \in R : -1 \leq x \leq 1, x \neq 0\}$  is  
 (A) neither Hausdorff nor first countable  
 (B) Hausdorff but not first countable  
 (C) first countable but not Hausdorff  
 (D) both Hausdorff and first countable
- Q. 78 Suppose  $U = \{x \in Q : 0 \leq x \leq 1\}$  and  $V = \{x \in Q : 0 < x < 2\}$ . Let  $n$  and  $m$  be the number of connected components of  $U$  and  $V$  respectively. Then  
 (A)  $m = n = 1$  (B)  $m = n \neq 1$   
 (C)  $m = 2n, m, n$  finite (D)  $m > 2n$
- Q. 79 Let  $f : [0, 1] \rightarrow R$  be the continuous function defined by
- $$f(x) = \frac{(x-1)(x-2)}{(x-3)(x-4)}.$$
- Then the maximal subset of  $R$  on which  $f$  has a continuous extension is  
 (A)  $(-\infty, 3)$  (B)  $(-\infty, 3) \cup (4, \infty)$   
 (C)  $R \setminus \{3, 4\}$  (D)  $R$
- Q. 80 Suppose  $U = (0, 1/2) \times (0, 1/2)$ ,  $V = (-1/2, 0) \times (-1/2, 0)$  and  $D$  be the open unit disk with centre at origin of  $R^2$ . Let  $f$  be a real valued continuous function on  $D$  such that  $f(U) = 0$ . Then it follows that  
 (A)  $f(v) = 0$  for every  $v$  in  $V$  (B)  $f(v) \neq 0$  for every  $v$  in  $V$   
 (C)  $f(v) = 0$  for some  $v$  in  $V$  (D)  $f$  can assume any real value on  $V$

- Q. 81 Suppose  $X$  is a random variable and  $f, g: R \rightarrow R$  are measurable functions such that  $f(X)$  and  $g(X)$  are independent, then  
 (A)  $X$  is degenerate  
 (B) both  $f(X)$  and  $g(X)$  is degenerate  
 (C) either  $f(X)$  or  $g(X)$  is degenerate  
 (D)  $X, f(X)$  and  $g(X)$  could all be non-degenerate
- Q. 82 Suppose  $X_1, X_2, \dots, X_n$  is a random sample from a  $N(\mu, \sigma^2)$  distribution, where  $\mu$  is known, but  $\sigma^2$  is not. If  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$  and  $S = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \mu)^2}$ , then the pair  $(\bar{X}, S)$  is  
 (A) complete and sufficient (B) complete but not sufficient  
 (C) sufficient but not complete (D) neither sufficient nor complete
- Q. 83 If  $X$  and  $Y$  are random variables with  $0 < \text{Var}(X), \text{Var}(Y) < \infty$ , consider the statements : (I)  $\text{Var}(E(Y/X)) = \text{Var}(Y)$  and (II) the correlation co-efficient between  $X$  and  $Y$  is  $\pm 1$ . Then  
 (A) (I) implies (II) and (II) implies (I)  
 (B) (I) implies (II) but (II) does not imply (I)  
 (C) (II) implies (I) but (I) does not imply (II)  
 (D) neither does (I) imply (II) nor does (II) imply (I)
- Q. 84 If the random variable  $X$  has a Poisson distribution with parameter  $\lambda$  and the parametric space has three elements 3, 4 and  $k$ , then to test the null hypothesis  $H_0: \lambda = 3$  vs. the alternative hypothesis  $H_1: \lambda \neq 3$ , a uniformly most powerful test at any level  $\alpha \in (0, 1)$  exists for any sample size  
 (A) for all  $k \neq 3, 4$  (B) if and only if  $k > 4$   
 (C) if and only if  $k < 3$  (D) if and only if  $k > 3$
- Q. 85 Suppose the random variable  $X$  has a uniform distribution  $P_\theta$  in the interval  $[\theta - 1, \theta + 1]$ , where  $\theta \in Z$ . If a random sample of size  $n$  is drawn from this distribution, then  $P_\theta$  almost surely for all  $\theta \in Z$ , a maximum likelihood estimator (MLE) for  $\theta$   
 (A) exists and is unique  
 (B) exists but may or may not be unique  
 (C) exists but cannot be unique  
 (D) does not exist
- Q. 86 A  $\chi^2$  (chi-squared) test for independence between two attributes  $X$  and  $Y$  is carried out at 2.5% level of significance on the following  $2 \times 2$  contingency table showing frequencies

$Y \backslash X$	$X_1$	$X_2$
$Y_1$	1	0
$Y_2$	1	d

If the upper 2.5% point of the  $\chi_1^2$  distribution is given as 5.0, then the hypothesis of independence is to be rejected if and only if

- (A)  $d > 1$  (B)  $d > 3$   
 (C)  $d > 5$  (D)  $d > 9$

- Q. 87 Consider the Linear Programming Problem (LPP) :  
 Maximize  $x_1$ ,  
 subject to :  $3x_1 + 4x_2 \leq 10$ ,  $5x_1 - x_2 \leq 9$ ,  $3x_1 - 2x_2 \geq -2$ ,  $x_1 - 3x_2 \leq 3$ ,  
 $x_1, x_2 \geq 0$ . The value of the LPP is

(A)  $\frac{9}{5}$  (B) 2

(C) 3 (D)  $\frac{10}{3}$

- Q. 88 Given that the eigenvalues of the integral equation

$$y(x) = l \int_0^{2\pi} \cos(x+t)y(t) dt$$

are  $\frac{1}{\pi}$  and  $-\frac{1}{\pi}$  with respective eigen functions  $\cos x$  and  $\sin x$ . Then the integral equation

$$y(x) = \sin x + \cos x + \lambda \int_0^{2\pi} \cos(x+t)y(t) dt \text{ has}$$

(A) unique solution for  $\lambda = 1/\pi$  (B) unique solution for  $\lambda = -1/\pi$

(C) unique solution for  $\lambda = \pi$  (D) no solution for  $\lambda = -\pi$

- Q. 89 The values of  $\lambda$  for which the integral equation

$$y(x) = \lambda \int_0^1 (6x-t)y(t) dt$$

has a nontrivial solution, are given by the roots of the equation

(A)  $(3\lambda - 1)(2 + \lambda) - \lambda^2 = 0$  (B)  $(3\lambda - 1)(2 + \lambda) + 2 = 0$

(C)  $(3\lambda - 1)(2 + \lambda) - 1\lambda^2 = 0$  (D)  $(3\lambda - 1)(2 + \lambda) + \lambda^3 = 0$

- Q. 90 The extremals for the functional

$$v[y(x)] = \int_{x_0}^{x_1} (xy' + y'2) dx$$

are given by the following family of curves :

(A)  $y = c_1 + c_2x + \left(\frac{x^2}{4}\right)$  (B)  $y = 1 + c_1x + c_2\left(\frac{x^2}{4}\right)$

(C)  $y = c_1 + x + c_2\left(\frac{x^2}{4}\right)$  (D)  $y = c_1 + c_2x - \left(\frac{x^2}{4}\right)$

**END OF THE QUESTION PAPER**